On the role of electron-phonon interaction in the resistance anisotropy of two-dimensional electrons in GaAs heterostructures

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Abstract. A contribution of the electron-phonon interaction into the energy of a unidirectional charge ordered state (stripe phase) of two-dimensional electrons in GaAs heterostructures is analyzed. The dependence of the energy on the direction of the electron density modulation is calculated. It is shown that in electrons layers situated close to the (001) surface the interference between the piezoelectric and the deformation potential interaction causes a preferential orientation of the stripes along the [110] axis.

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1. Introduction

The observation of a resistance anisotropy of two-dimensional (2D) electrons at high Landau level occupancy [1, 2, 3, 4] is considered as an evidence for a formation of unidirectional charge ordered states (stripe phases) in such systems. These states were predicted in Refs. [5, 6, 7]. The Hartree-Fock calculations [5, 6, 7] show that electrons at a topmost half-filled Landau level are separated in stripes of full and empty occupancy with a period of about several times the cyclotron radius. If the stripes are preferentially oriented along a certain direction, such states are expected to demonstrate the anisotropy in the longitudinal resistance: the low resistance along the stripes and the high resistance at the perpendicular direction [8, 9].

The effect is observed in 2D electron layers in GaAs/AlGaAs heterostructures grown on (001)-oriented GaAs substrates. For such an experimental setup in the magnetic field perpendicular to the electron layer the low and high resistance directions are always pinned along certain crystallography axes of the host matrix, namely, [110] and $[1\bar{1}0]$, correspondingly.

A mechanism that determines a preferential orientation of the stripes in GaAs heterostructures is not completely understood yet. It was suggested by Takhtamirov and Volkov [10], that an anisotropy of the effective mass of 2D electrons may be responsible for the orientational pinning of the stripes. In the model developed in Ref. [10] the

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effective mass anisotropy is caused by an asymmetry of the quantum well potential confining the electrons to the (001) plane. This idea has been checked experimentally by Cooper *et al* [11]. In experiments [11] the influence of the symmetry of the quantum well potential on the resistance anisotropy has been studied. It has been found, that the resistance does not depend on the precise form of the confining potential, in particular, the same form of the anisotropy is observed for symmetrically confining 2D electrons. The authors of Ref. [11] conclude that the asymmetry of the confining potential cannot be considered as an important factor.

Another possible mechanism for the stripe orientation has been studies by the present author [12]. It was shown, that the piezoelectric interaction lowers the energy of the charge density wave aligned along either the [110] or [1$\bar{1}$0] axes. Nevertheless, the piezoelectric mechanism alone does not explain why only one of two preferential orientations is realized. In this paper we address this problem. We extend the model proposed in [12] and take into account, together with the piezoelectric interaction, the deformation potential interaction. We find that, for the electron layers situated near the (001) surface of the sample, an interference between two channels of the electron-phonon coupling plays an important role in the stripe orientation. The mechanism considered explains the resistance anisotropy observed in the experiments.

2. The model

Let us consider a semi-infinite piezoelectric crystal that occupies a volume $z < 0$ and contains an electron layer at the distance d from the surface. The static energy of the system can be presented in the form

$$U = \int_{z<0} d^3r(\frac{\mathbf{ED}}{8\pi} + \frac{u_{ik}\sigma_{ik}}{2}) + \int_{z>0} d^3r\frac{E^2}{8\pi} + U_{def} \ , \tag{1}$$

where

$$D_i = \varepsilon E_i - 4\pi\beta_{i,kl}u_{kl} \tag{2}$$

is the electric displacement field,

$$\sigma_{ik} = \lambda_{iklm}u_{lm} + \beta_{l,ik}E_l \ , \tag{3}$$

the stress tensor, $\mathbf{E}$, the electric field, u_{ik}, the strain tensor, λ_{iklm}, the elastic moduli tensor, $\beta_{i,kl}$, the piezoelectric moduli tensor, ε, the dielectric constant. To be more specific, we restrict our consideration to the case of a cubic lattice. The last term in (1) is the deformation potential interaction. It is chosen in the form

$$U_{def} = \int d^3r\Lambda\rho(u_{xx} + u_{yy})\delta(z + d) \ , \tag{4}$$

where Λ is the deformation potential constant, ρ, the 2D electron density. Since we consider the model of an electron layer of zero-thickness, the interaction with u_{zz} deformations is not included in (4).

The electric and elastic fields in Eq. (1) satisfy the following equations:

$$\nabla\mathbf{D} = 0 \ , \tag{5}$$

$$\frac{\partial \sigma_{ik}}{\partial x_k} = 0 \tag{6}$$

(at $z < 0$), and

$$\nabla \mathbf{E} = 0 \tag{7}$$

(at $z > 0$).

At the free surface ($z = 0$) the boundary conditions are

$$D_z\Big|_{z=-0} = E_z\Big|_{z=+0} , \tag{8}$$

$$E_{x(y)}\Big|_{z=-0} = E_{x(y)}\Big|_{z=+0} , \tag{9}$$

$$\sigma_{iz}\Big|_{z=-0} = 0 . \tag{10}$$

At $z = -d$ the normal component of the electric displacement field is discontinuous

$$D_z\Big|_{z=-d+0} - D_z\Big|_{z=-d-0} = 4\pi e\rho . \tag{11}$$

The deformation potential interaction induces a tangential force applied to the medium in the $z = -d$ plane. Rewriting the energy (4) in the form

$$U_{def} = -\int d^2r \sum_{i=x,y} u_i \Lambda \partial_i \rho , \tag{12}$$

where $\mathbf{u}$ is the displacement vector, we obtain the following expression for the force, applied to the unit area:

$$\mathbf{F} = \Lambda(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y}, 0) . \tag{13}$$

In equilibrium, this force is compensated by the stresses, and at $z = -d$ the stress tensor satisfies the boundary condition

$$\sigma_{iz}\Big|_{z=-d+0} - \sigma_{iz}\Big|_{z=-d-0} = -F_i . \tag{14}$$

Using equations (5,6,7) and the boundary conditions one can reduce the energy (1) to the form

$$U = \frac{1}{2}\int d^2r (e\rho\varphi - u_i F_i) , \tag{15}$$

where the electrostatic potential φ and the displacement field $\mathbf{u}$ are taken at $z = -d$. Their values are found from the solution of Eqs. (5-7) at the boundary conditions specified.

3. The stripe state energy anisotropy

Let us calculate the energy (15) for the stripe phase. We consider the system with the electron density modulated in a certain direction $\mathbf{r}_s$. The electron density can be presented as a Fourier series

$$\rho(\mathbf{r}_{pl}) = \sum_{G_n} e^{i\mathbf{G}_n\mathbf{r}_{pl}}\rho_{\mathbf{G}_n} \ , \tag{16}$$

where $G_n = n\mathbf{q}$ (n is integer), $\mathbf{q} \parallel \mathbf{r}_s$, $|q| = 2\pi/l$, and l is the period of the stripe structure. For simplicity, we analyze the case of a unimodal charge density wave

$$\rho(\mathbf{r}_{pl}) = \rho_0 \cos \mathbf{q}\mathbf{r}_{pl} \ . \tag{17}$$

We calculate the energy (15) as a series in powers of the electron-phonon interaction constants:

$$\frac{U}{S} = U_0 + U_2 + \dots \quad , \tag{18}$$

where S is the area of the electron layer,

$$U_0 = \frac{\pi e^2\rho_0^2}{2q\varepsilon}\left(1 + \frac{\varepsilon - 1}{\varepsilon + 1}e^{-2qd}\right) \ , \tag{19}$$

the Coulomb energy in the absence the electron-phonon interaction, and the term U_2 is quadratic in the interaction constants. For concrete physical systems considered below the contribution of the higher order terms in (18) is very small and it can be neglected. The term U_2 determines the dependence of the energy on the direction of $\mathbf{q}$.

Let us first consider an isotropic crystal, for which the sound velocities do not depend on the direction of the sound propagation. For a cubic lattice this condition is realized if the elastic constants satisfy the relation $c_{11} - c_{12} - 2c_{44} = 0$. For such a special case an analytical expression for U_2 can be presented in a simple form. For the (001) electron layer the calculation of U_2 gives the following result:

$$U_2 = A + B\cos 4\psi + C\sin 2\psi \ , \tag{20}$$

where ψ is the angle between $\mathbf{q}$ and the [100] axis. The angle dependence (20) is determined by the parameters $\eta = c_{11}/c_{44}$ and $\xi = qd$. Using the strong inequality $\epsilon \gg 1$, that takes place in $GaAs$, we find for B and C the following approximate expressions

$$B = E_p\left[1 - \frac{\eta}{3} - e^{-2\xi}\Big(\frac{(\eta-3)[2\eta(1-2\xi) + \xi^2(5\eta-3)]}{9(\eta-1)} + \xi^3[\frac{2}{3}(\eta+1) - \xi(\eta-1)]\Big)\right] , \tag{21}$$

$$C = E_i\ e^{-2\xi}\Big(\frac{\eta(\eta-3)(1-\xi)}{\eta-1} + \xi^2[3\xi(\eta-1) - 2\eta - 3]\Big) \ , \tag{22}$$

where $E_p = 9\pi^2e^2\rho_0^2e_{14}^2/64\varepsilon^2c_{11}q$, $E_i = \pi|e|\rho_0^2e_{14}\Lambda/8\varepsilon c_{11}$. We do not present here the expression for the parameter A which does not influence the orientation.

In Eq. (20) the second term describes the anisotropy determined by the piezoelectric interaction, and the third term - the anisotropy caused by the interference of the piezoelectric and the deformation potential interaction. One can see that at $\xi \to \infty$ the

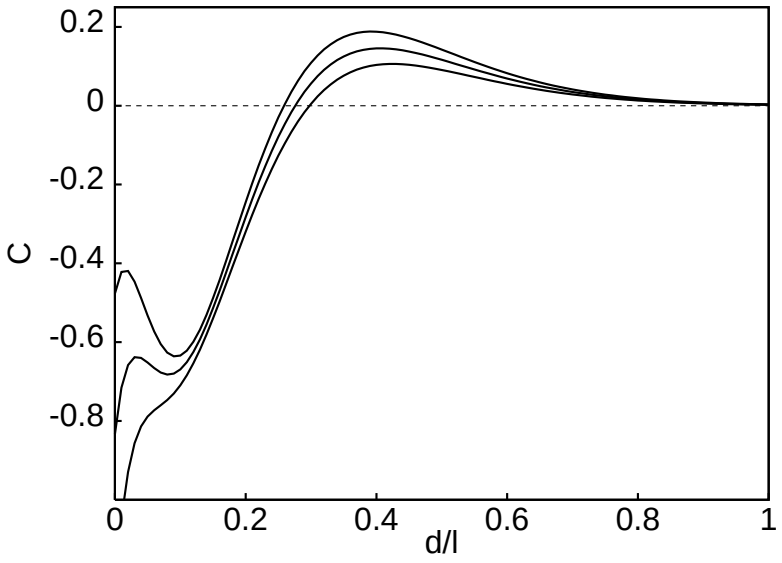

Figure 1. The parameter of the stripe energy anisotropy (Eq.(22)) in units of E_i for the isotropic crystal; $\eta = 2.7,\ 2.5,\ 2.3$ (from top to bottom).

interference term tends to zero and the energy (20) obeys the C_{4v} symmetry. If $\xi \sim 1$ the interference term is essential and the C_{4v} symmetry is reduced to the C_{2v} one. We find that at $\eta < 2.7$ the parameter B is positive for all ξ and the global minimum is reached at $\psi_m = \pi/4$ or $\psi_m = 3\pi/4$ depending on the sign of the parameter C. At $C < 0$ $\psi_m = \pi/4$ and the stripes are preferentially oriented along the $[1\bar{1}0]$ axis, while at $C > 0$ $\psi_m = 3\pi/4$ and the [110] oriented stripe phase has the lowest energy. The dependence of C on $d/l = \xi/2\pi$ is shown in Fig. 1. One can see that at $d/l > 0.3$ the [110] orientation of the stripes is realized

The results obtained are sensitive to the parameters of the system. Therefore, to determine the stripe orientation in GaAs heterostructures it is necessary to take into account the anisotropy of the elastic moduli. For this case we solve Eqs. (5-7), with the boundary conditions specified, numerically. The following parameters are used for the calculations: $c_{11} = 12.3 \cdot 10^{10}\ N/m^2$, $c_{12} = 5.4 \cdot 10^{10}\ N/m^2$, $c_{44} = 6.0 \cdot 10^{10}\ N/m^2$, $e_{14} = 0.15\ C/m^2$ $\Lambda = 7.4\ eV$, $\varepsilon = 12.5$. The dependence of U_2 on the direction of $\mathbf{q}$ at $l = 2 \cdot 10^3$ Å and several d/l is shown in Fig. 2. In difference with the isotropic crystal, the minima of the energy are reached at q deviated from the [110] or $[1\bar{1}0]$ axes to the angles $\Delta\psi \approx \pm\pi/12$. If d/l is in the interval [0.23,1], the configurations with a small deviation of q from the $[1\bar{1}0]$ axis have the lowest energy. In this case one can expect that a domain structure is formed, and, in average, the system should demonstrate the minimum resistance in the [110] direction and the maximum resistance

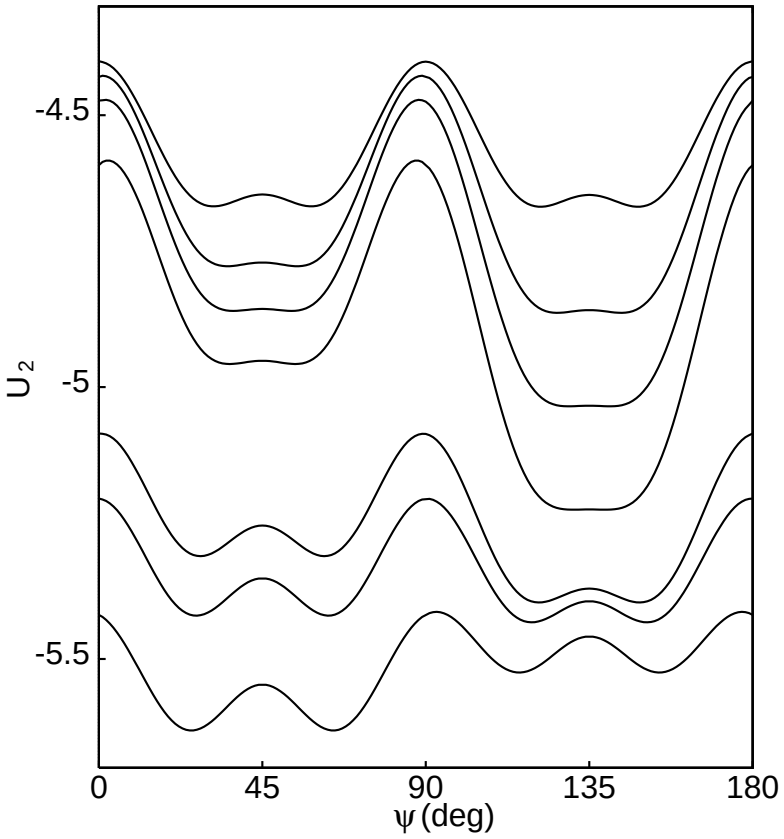

Figure 2. The phonon contribution to the energy of the stripe structure (in units of E_p) for GaAs versus the direction of the electron density modulation at $d/l = 1.0,\ 0.6,\ 0.5,\ 0.4,\ 0.25, 0.23,\ 0.2$ (from top to bottom).

in the perpendicular direction. The calculations predict the largest resistance anisotropy at $d/l = 0.4$. At larger and at smaller d/l the anisotropy becomes weaker. At $d/l > 1$ and $d/l \approx 0.23$ it disappears completely. At $d/l < 0.23$ the resistance anisotropy resets, but the high and the low resistance directions alternates.

4. Discussion

We have shown that the classical energy of a charge density wave in a 2D electron system embedded in a piezoelectric matrix depends on the direction of the wave vector. The effect is caused by the electron-phonon interaction. The minimum energy is reached at two or four different directions of the density modulation. Therefore, the stripes may form a polydomain structure. In the bulk samples polydomain structures may not show

any resistance anisotropy, while in 2D layers situated near the surface of the sample the resistance anisotropy should take place.

We calculate the static energy of the stripe structure. Our results can be also understood as an effect of a virtual exchange of acoustic phonons between electrons. The electron-electron interaction in bulk isotropic piezoelectrics caused by the virtual exchange of acoustic phonons has been studied by Rashba and Sherman [13]. Our approach [12] (see also Ref. [14], where the orientation of bi-layer Wigner crystals has been studied) reproduces the results of Ref. [13]. Within such an interpretation the effect described in the present paper is a consequence of the virtual exchange by surface phonon modes. For the surface acoustic waves on the (001) surface the piezoelectric and deformation potential interactions give in-phase contributions to the matrix elements of the electron-phonon interaction [15], and the interference between two channel of electron-phonon interaction takes place.

The unimodal approximation predicts that at $d/l > 0.23$ the monodomain stripe structure has the lowest energy for the stripes deviated from the [110] axis to the angle $\Delta\psi \approx +\pi/12$ or $\Delta\psi \approx -\pi/12$. In this case, the polydomain structure should demonstrate an anisotropic resistance with a minimum at the [110] direction. The largest resistance anisotropy is reached at $d/l \approx 0.4$. At $d/l > 1$ the anisotropy becomes exponentially weak. At $d/l < 0.23$ the anisotropy changes its sign. Note, that the last conclusion is specific for the unimodal approximation (17) only. The higher harmonics in (16) will shift the transition point to a smaller value of d/l.

We consider our model describes the orientation of stripe structures formed at high Landau levels. We analyze the anisotropy of the direct interaction between electrons. One can expect that the anisotropy of the exchange interaction is small, and it does not influence significantly the orientation.

In the Hartree-Fock approximation the period of the electron density modulation l is approximately equal to $6\ell_H$, where ℓ_H is the magnetic length. In experiments [11] the resistance anisotropy was observed at the magnetic field $H \approx 2$ T and for $d \approx 2 \cdot 10^3$ Å. Therefore, for the Hartree-Fork l we find the ratio $d/l \approx 2$. At such d/l the surface effects are not important and only an exponentially small violation of the C_{4v} symmetry may take place. Since in experiments this violation is quite large we suppose that the period l is large then it follows from the Hartree-Fock theory.

It is interesting to evaluate the absolute value of the native anisotropy caused by the phonon mechanism. Using the parameters given before for $d/l = 0.4$, the filling factor $\nu = 9/2$, the electron density $n = 2 \cdot 10^{11}$ cm^{-2}, and $\rho_0 \approx \bar{\rho}$ (where $\bar{\rho}$ is the average density at the valence Landau level) we find the anisotropy energy $E_a \approx 0.7$ mK per electron (we determine E_a as the energy at $\mathbf{q} \parallel [110]$ minus the energy at $\mathbf{q} \parallel [1\bar{1}0]$).

In our consideration we neglect the screening of the electron-phonon coupling caused by the polarization of the remote Landau levels. To evaluate the effect of screening in the unimodal approximation one can use the effective dielectric constant $\varepsilon_{eff}(q) = \varepsilon(1+K_q)$ and the screened deformation potential $\Lambda_{scr}(q) = \Lambda/(1+K_q)$, given by the random phase

approximation. Here

$$K_q = \frac{e^2 q}{\varepsilon \omega_c} \exp\left(-\frac{q^2 \ell_H^2}{2}\right) \sum_{n,m,\sigma}{}' \frac{n!}{m!} \frac{f_{n\sigma}(1-f_{m\sigma})}{m-n} \left(\frac{q^2\ell_H^2}{2}\right)^{m-n-1} \left[L_n^{m-n}\left(\frac{q^2\ell_H^2}{2}\right)\right]^2. \qquad (23)$$

In Eq. (23) $f_{n\sigma} = n_F(\varepsilon_{n\sigma})$ is the Fermi factor, ω_c is the cyclotron frequency, L_n^m is the generalized Laguerre polynomial, and the prime on the sum excludes the valence Landau level (compare with Ref. [16]). Under such a substitution, the form of the dependence $U_2(\psi)$ (Fig. 2) remains almost unchanged, but the absolute value of the anisotropy is reduced as $E_a^{src} \approx E_a/(1+K_q)^2$. Evaluation of the formula (23) for the parameters given above yields $E_a^{src} \approx 0.4$ mK.

The phonon contribution to the native anisotropy is comparable with the one given by the effective mass anisotropy mechanism [10] (it gives the value of order of 1 mK). The experimental data for the anisotropy energy have been obtained in Ref. [11] from the measurements of the resistivity in a tilted magnetic field. The data presented in [11] are based on theoretical calculations by Jungwirth *et al* [16]. According to Ref.[11] the anisotropy energy for the sample with a conventional heterostructure is higher (2.4 mK), then for the sample with a symmetric quantum well (0.5 mK). Thus, we conclude that two mechanisms of the anisotropy work in parallel, and the survival of the anisotropy in samples with symmetric quantum wells can be accounted for the phonon mechanism.

The mechanism of the resistance anisotropy considered is this paper is essentially dependent on the distance between the surface of the sample and the electron layer. Therefore, it is desirable to investigate this dependence experimentally. Such a study may answer the question whether or not the surface effects play an important role in the stripe orientation.

It is also of interest to investigate experimentally the influence of the electron layer orientation on the resistance anisotropy. To illustrate this point, we outline the results obtained for the (111) layer in the isotropic crystal. For such a system the energy (20) is modified to

$$u_2 = A' + B' \cos 6\psi \qquad (24)$$

where ψ is counted from the $[0\bar{1}1]$ axis. The coefficient B' is given by the following expression:

$$B' = E_p \frac{10}{27}\Big(1-\eta - \frac{e^{-2\xi}}{5(\eta-1)}[4(\eta^2-\eta-1)(1+2\xi) + \xi^2(\eta+1)(5\eta-7) + \frac{2}{3}\xi^3(\eta+1)(\eta-5) - \xi^4(\eta-1)^2]\Big) \qquad (25)$$

It is important to note, that the interference term in (24) does not depend on ψ (it is included in A'). We find that the parameter B' is negative for all ξ, and $\eta > 2$. Therefore, the minimum of the energy is reached at $\psi_m = n\pi/3$ (n is integer). It means, that monodomain stripe structures should demonstrate the low resistance along any of the $[\bar{2}11]$, $[1\bar{2}1]$,or $[11\bar{2}]$ direction, while the polydomain structures may not show any resistance anisotropy at all. Since this conclusion is not sensitive to the parameters

of the systems, the lattice anisotropy is not very essential in this case, and the same behavior is expected for the GaAs system.

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